$$= \frac{1}{2n+1}$$

Donc

$$\forall n \in \mathbb{N}^*, \quad \boxed{I_{n+1} + I_n = \frac{1}{2n+1}}$$

b Calculer alors: $J = \int_0^1 \frac{7x^8 + 7x^6 - 3x^4 + 4}{x^2+1} dx$

$$J = \int_0^1 \frac{7u^8 + 7u^6 - 3u^4 + 4}{u^2+1} du$$

$$= 7\left(\int_0^1 \underbrace{\frac{u^8}{1+u^2}}_{I_4} dx + \int_0^1 \underbrace{\frac{u^6}{1+u^2}}_{I_3} dx \right)$$

$$- 3 \cdot \int_0^1 \underbrace{\frac{u^4}{1+u^2}}_{I_2} dx + 4 \cdot \int_0^1 \underbrace{\frac{1}{1+u^2}}_{J_0} dx$$

Or

$$I_n = \int_0^1 \frac{u^{2n}}{1+u^2} dx$$

$$J = 7\left(I_4 + I_3 \right) - 3 \cdot I_2 + 4 \cdot I_0$$

$$= 7\left(\boxed{I_4 + I_3} \right) - 3\left(I_2 + I_1 \right) + 3I_1 + 4I_0$$